A number m is randomly selected from the set $\{11, 13, 15, 17, 19\}$ and a number n is randomly selected from $\{1999, \ldots, 2018\}$. What is the probability that m^n has a units digit of 1?

For 11: $11^n \equiv 1 \mod 10$ for all n.

(probability = 1)

For 13:
$$13^1 \equiv 3 \mod 10$$
$$13^2 \equiv 9 \mod 10$$
$$13^3 \equiv 7 \mod 10$$
$$13^4 \equiv 1 \mod 10$$

In the second set, we have

$$1999 \equiv 3 \mod 4$$
$$2016 \equiv 0 \mod 4$$
$$2017 \equiv 1 \mod 4$$
$$2018 \equiv 2 \mod 4$$

1999
2000 - 2003
2004 - 2007
2008 - 2011
2012 - 2015
2016
2017
2018

So the probability that a randomly selected element of the second set is congruent to 0 mod 4 is $\frac{1}{4}$.

<u>For 15</u>: $15^n \equiv 5 \mod 10$ For all n

(probability = 0)

<u>For 17</u>: $17^1 \equiv 7 \mod 10$

$17^2 \equiv 9 \mod 10$

$17^3 \equiv 3 \mod 10$

$17^4 \equiv 1 \mod 10$

So probability = $\frac{1}{4}$

<u>For 19</u>: $19^1 \equiv 9 \mod 10$

$19^2 \equiv 1 \mod 10$

So probability = $\frac{1}{2}$

$\therefore$ P(units digit of m^n is 1)

$= 1 \times \frac{1}{5} + \frac{1}{4} \times \frac{1}{5} + 0 \times \frac{1}{5} + \frac{1}{4} \times \frac{1}{5} + \frac{1}{2} \times \frac{1}{5}$

$= \frac{2}{5}$