

A rectangle with positive integer side lengths in cm has area A cm² and perimeter P cm. Which of the following numbers cannot equal $A + P$?

- (A) 100 (B) 102 (C) 104 (D) 106 (E) 108

$$xy = A$$

$$2(x+y) = P$$



$$A + P = xy + 2(x+y)$$

First trial & error:

$$\begin{aligned} 8 \times 8 + 2(8+8) &= 64 + 32 \\ &= 96 \end{aligned}$$

$$\begin{aligned} 8 \times 9 + 2(8+9) &= 72 + 2(17) \\ &= 72 + 34 \\ &= \textcircled{106} \end{aligned}$$

$$\begin{aligned} 7 \times 10 + 2(7+10) &= 70 + 34 \\ &= \textcircled{104} \end{aligned}$$

$$\begin{aligned} 11 \times 6 + 2(11+6) &= 66 + 2(17) \\ &= 66 + 34 \\ &= \boxed{100} \end{aligned}$$

$$\begin{aligned} 12 \times 6 + 2(12+6) &= 72 + 2(18) \\ &= 72 + 36 \\ &= \boxed{108} \end{aligned}$$

$$\begin{aligned} 15 \times 5 + 2(15+5) &= 75 + 2(20) \\ &= 75 + 40 \\ &= 115 \end{aligned}$$

$$\begin{aligned} 15 \times 4 + 2(15+4) &= 60 + 2(19) \\ &= 60 + 38 \\ &= 98 \end{aligned}$$

$$\begin{aligned} 14 \times 5 + 2(14+5) &= 70 + 38 \\ &= \boxed{108} \end{aligned}$$

$$\begin{aligned} 16 \times 4 + 2(16+4) &= 64 + 2 \times 20 \\ &= 104 \end{aligned}$$

$$\begin{aligned}17 \times 4 + 2(17+4) &= 68 + 2(21) \\ &= 68 + 42 \\ &= \boxed{110}\end{aligned}$$

So the answer is 102

Now an algebraic solution:

$$\begin{aligned}A+P &= xy + 2x + 2y \\ &= \frac{(x+2)(y+2) - 4}{xy + 2y + 2x + 4}\end{aligned}$$

For 102:

$$\begin{aligned}102 + 4 &= 106 \\ &= 2 \times 53 \\ &= (0+2)(51+2)\end{aligned}$$

but 0 is not
a positive integer

\therefore 102 not a valid choice

For $A+P$