For how many integers n is

$$\frac{n}{20-n}$$

a square number?

We have $\frac{n}{20-n} = k^2$

$$\Rightarrow n = k^2(20-n)$$

$$= 20k^2 - k^2 n$$

$$\Rightarrow n + k^2 n = 20k^2$$

$$\Rightarrow n(1+k^2) = 20k^2$$

$$\Rightarrow n = \frac{20k^2}{1+k^2}$$

Now, $(1+k^2)$ is <u>not</u> a Factor of k^2, so For n to be an integer, $1+k^2$ must be a Factor of 20.

$1+k^2$	k^2	Square?
1	0	yes
2	1	yes
4	3	no

5	4	yes
10	9	yes
20	19	no

So there are (4) answers.