If $\theta$ is an acute angle and

$$\sin\left(\tfrac{1}{2}\theta\right) = \sqrt{\frac{x-1}{2x}}$$

Find $\tan(\theta)$

We have $\cos\left(\tfrac{1}{2}\theta\right) = \sqrt{1-\sin^2\left(\tfrac{1}{2}\theta\right)}$

$$= \sqrt{1 - \frac{x-1}{2x}}$$

$$= \sqrt{\frac{2x-(x-1)}{2x}}$$

$$= \sqrt{\frac{x+1}{2x}}$$

and so $\tan\left(\frac{\theta}{2}\right) = \dfrac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)}$

$$= \sqrt{\frac{x-1}{2x}} \cdot \sqrt{\frac{2x}{x+1}}$$

$$= \sqrt{\frac{x-1}{x+1}}$$

Use the Double Angle Formula:

$$\tan(\theta) = \tan\left(2 \cdot \tfrac{1}{2}\theta\right)$$

$$= \frac{2\tan\left(\frac{1}{2}\theta\right)}{1-\tan^2\left(\frac{1}{2}\theta\right)}$$

$$= \frac{2\sqrt{\frac{x-1}{x+1}}}{1-\frac{x-1}{x+1}}$$

$$= \frac{2\sqrt{\frac{x-1}{x+1}}}{\left[\frac{x+1-(x-1)}{x+1}\right]}$$

$$= \frac{2\sqrt{\frac{x-1}{x+1}}}{\left[\frac{2}{x+1}\right]}$$

$$= \sqrt{\frac{x-1}{x+1}} \cdot x+1$$

$$= \sqrt{(x-1)(x+1)}$$

$$= \sqrt{x^2-1}$$