

$x \in \mathbb{R}$ . If  $\sec(x) - \tan(x) = 2$  then what is  $\sec(x) + \tan(x)$ ?

We will use the difference of squares:

$$\sec^2(x) - \tan^2(x) = (\sec(x) - \tan(x))(\sec(x) + \tan(x))$$

$$\Rightarrow \sec(x) + \tan(x) = \frac{\sec^2(x) - \tan^2(x)}{\sec(x) - \tan(x)}$$

$$= \frac{\sec^2(x) - \tan^2(x)}{2}$$

2 ← given in question

Now, we have

$$\sec^2(x) - \tan^2(x) = \frac{1}{\cos^2(x)} - \frac{\sin^2(x)}{\cos^2(x)} \quad \left| \begin{array}{l} \text{defns} \\ \text{of } \sec \\ \& \tan \end{array} \right.$$

$$= \frac{1 - \sin^2(x)}{\cos^2(x)} \quad \left| \begin{array}{l} \text{simplify} \\ \text{fractions} \end{array} \right.$$

$$= \frac{\cos^2(x)}{\cos^2(x)} \quad \left| \begin{array}{l} \text{Pythagorean} \\ \text{identity} \\ \sin^2(x) + \cos^2(x) = 1 \end{array} \right.$$

$$= 1$$

therefore  $\sec(x) + \tan(x) = \frac{1}{2}$