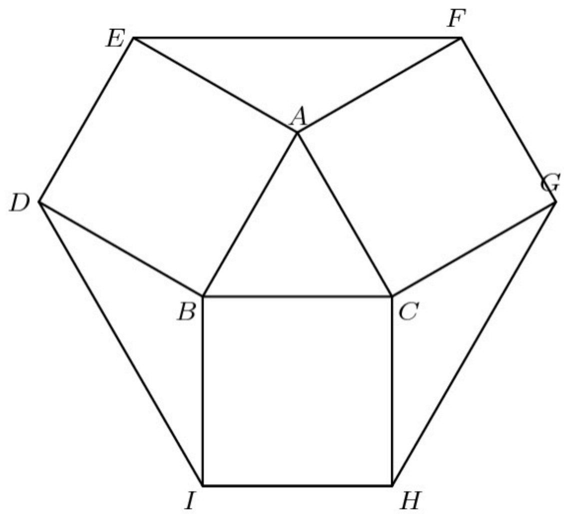
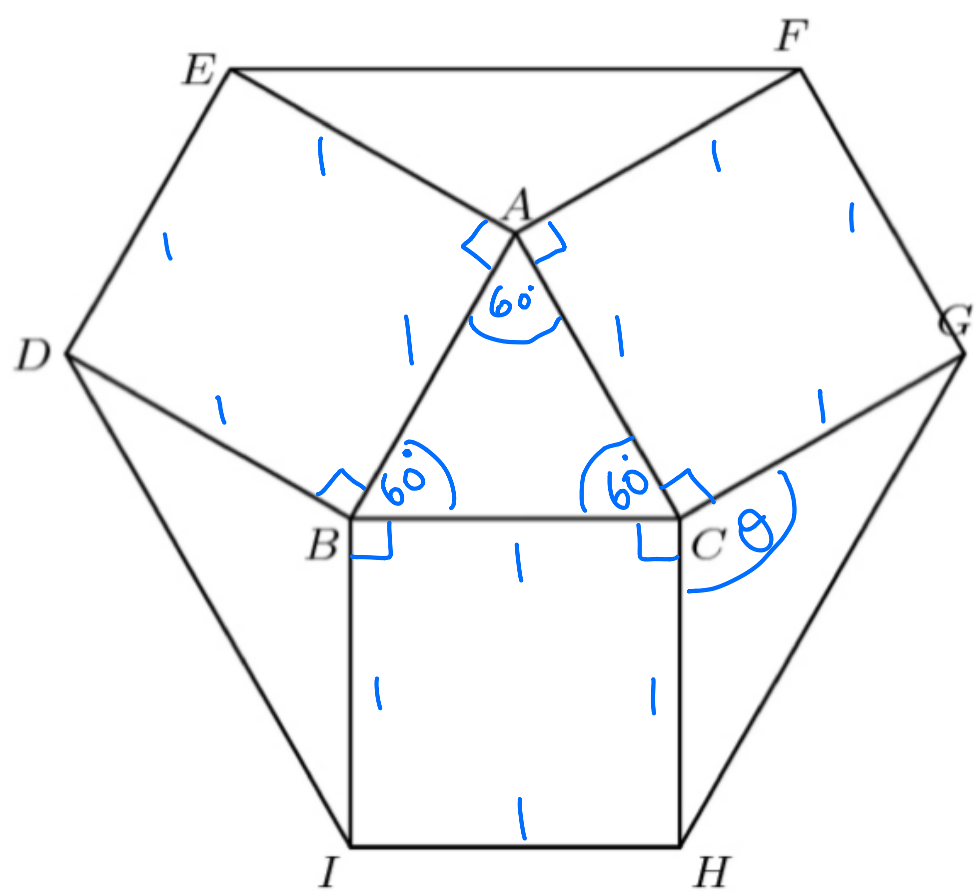
Equilateral $\triangle ABC$ has side length 1, and squares $ABDE$, $BCHI$, $CAFG$ lie outside the triangle. What is the area of hexagon $DEFGHI$?





We have

$$60^\circ + 2(90^\circ) + \theta = 360^\circ$$

(angle of revolution about C)

$\Rightarrow \quad 60^\circ + 180^\circ + \theta = 360^\circ$

$\Rightarrow \quad \theta = 120^\circ$

Using the Sine Area Formula, we have

$$\begin{aligned}
\text{Area}(\triangle CGH) &= \frac{1}{2} \cdot 1 \cdot 1 \cdot \sin(\theta) \\
&= \frac{1}{2} \sin(120^\circ) \\
&= \frac{1}{2} \left[2 \sin(60^\circ) \cos(60^\circ) \right] \\
&= \frac{\sqrt{3}}{2} \cdot \frac{1}{2} \\
&= \frac{\sqrt{3}}{4} \text{ units}^2
\end{aligned}$$

The triangles $\triangle DBI$ and $\triangle EAF$ are congruent to $\triangle CGH$ and therefore have equal area.

The area of each of the squares is $1 \times 1 = 1$ unit^2.

The area of the triangle $\triangle ABC$, again using the Sine Area Formula, is

$$\frac{1}{2} \times 1 \times 1 \times \sin(60^\circ) = \frac{1}{2} \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{4}$$

Hence the area of the hexagon is

$$(3 \times 1) + \left(4 \times \frac{\sqrt{3}}{4}\right) = 3 + \sqrt{3}$$

units^2