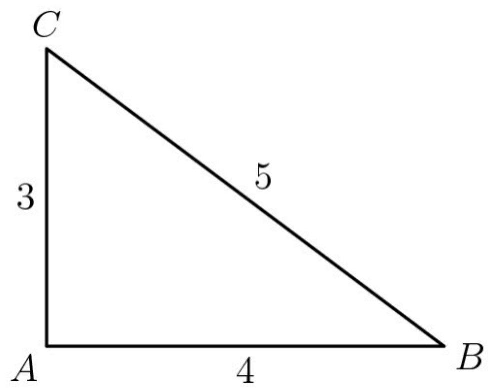
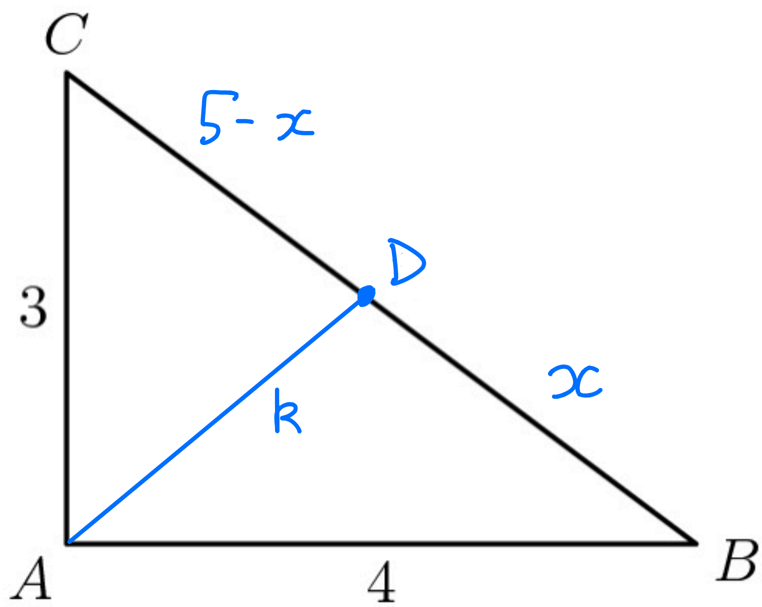
In the figure below, choose point D on $\overline{BC}$ so that $\triangle ACD$ and $\triangle ABD$ have equal perimeters. What is the area of $\triangle ABD$?



First let's draw the point D onto the diagram:



We have

$$\text{Perimeter}(\triangle ACD) = 3 + k + 5 - x$$
$$= 8 + k - x$$

and

$$\text{Perimeter}(\triangle ABD) = 4 + k + x$$

We are told that the two perimeters are equal, hence

$$8 + k - x = 4 + k + x$$

$$\Rightarrow \quad 4 = 2x$$

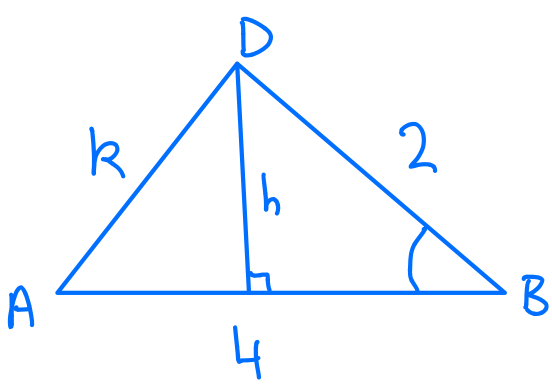
$$\Rightarrow \quad x = 2$$

In $\triangle ABC$, we have

$$\sin(\angle ABC) = \frac{3}{5}$$

But $\angle ABC = \angle ABD$ since B, C and D are collinear.

So we can use this information to work out the area of $\triangle ABD$ using the Sine Area Formula or by finding h below:



$$\sin(\angle ABD) = \frac{h}{2}$$

$$\Rightarrow \frac{3}{5} = \frac{h}{2}$$

$$\Rightarrow h = \frac{6}{5}$$

We have

$$\text{Area}(\angle ABD) = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 4 \times \frac{6}{5}$$

$$= \boxed{\frac{12}{5} \text{ units}^2}$$

Note: We never actually worked out what k was as it was unnecessary with regard to finding the area.