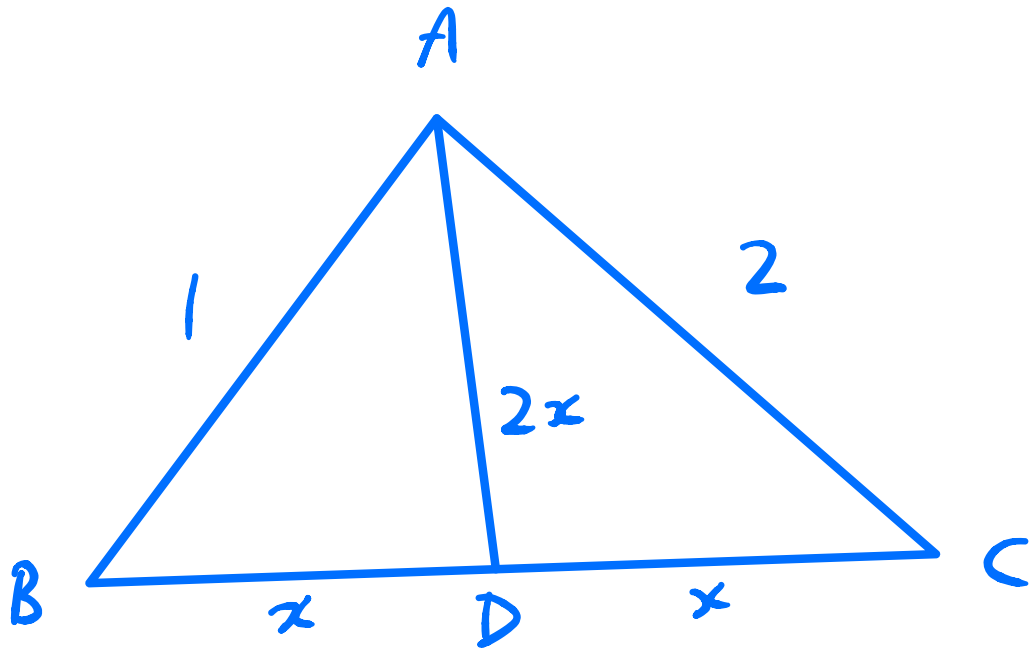
In $\triangle ABC$ we have $AB = 1$ and $AC = 2$ Side BC and the median From A to Bc have the same length. What is BC?



Let D be the midpoint of BC.

Cos rule in $\triangle ADC$:

$$(2x)^2 = x^2 + 2^2 - 2(x)(2)\cos(C)$$

$$\Rightarrow 4x^2 = x^2 + 4 - 4x\cos(C) \quad (1)$$

Cos rule in $\triangle ABC$:

$$1^2 = (2x)^2 + 2^2 - 2(2x)(2)\cos(C)$$

$$\Rightarrow \quad 1 = 4x^2 + 4 - 8x\cos(C) \quad (2)$$

Solve simultaneously to Find x:

$2 \times (1) - (2)$:

$$8x^2 - 1 = 2x^2 + 8 - 8x\cos(C)$$
$$- 4x^2 - 4 + 8x\cos(C)$$
$$= -2x^2 + 4$$

$$\Rightarrow \quad 10x^2 = 5$$

$$\Rightarrow \quad x^2 = \frac{1}{2}$$

$$\Rightarrow \quad x = \frac{1}{\sqrt{2}} \quad (\text{distance} \Rightarrow \text{take positive soln})$$

$$\Rightarrow BC = 2 \cdot \frac{1}{\sqrt{2}} = \boxed{\sqrt{2}}$$